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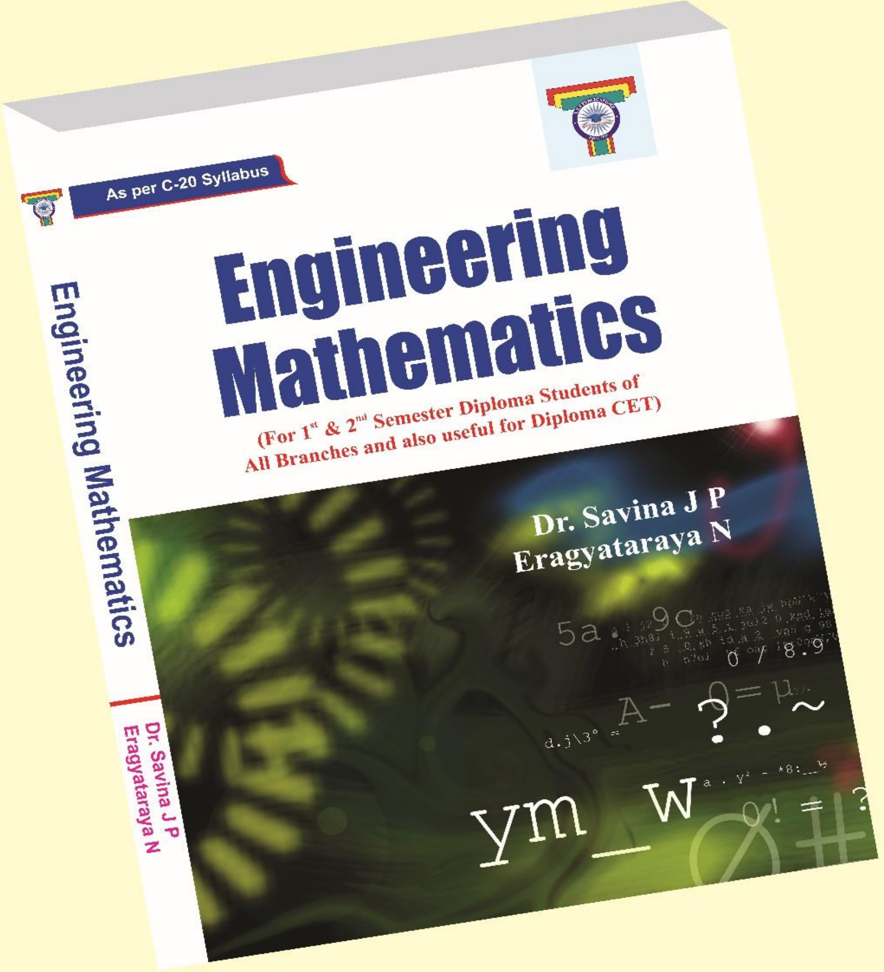

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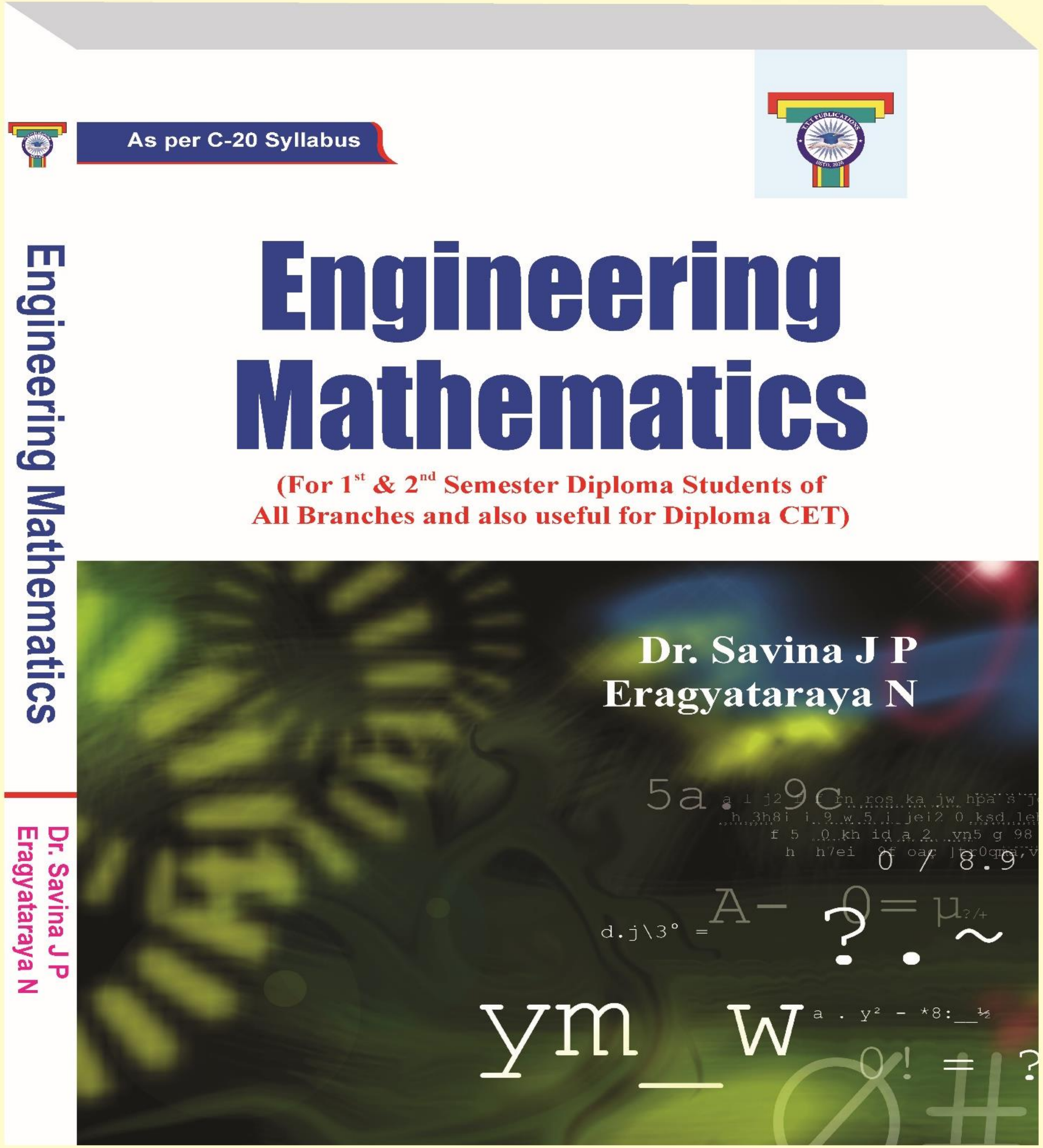

Book Front Page

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Engineering Mathematics

As per C-20 Syllabus

For 1st & 2nd Semester Diploma Students of All Branches and also useful for Diploma CET

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Syllabus

As per C-20 D.T.E. Syllabus

C-20 SYLLABUS

UNIT NO	Unit skill set (In cognitive domain)	Topics/Subtopics	Hours L-T-P
UNIT-1 MATRICES AND DETERMINANTS	➢ Use algebraic skills which are essential for the study of systems of linear equations, Matrix algebra and Eigen values	1.1 Matrix and types 1.2 Algebra of Matrices (addition, subtraction, scalar multiplication and multiplication) 1.3 Evaluation of determinants of a square matrix of order 2 and 3. Singular matrices 1.4 Cramer's rule for solving system of linear equations involving 2 and 3 variables 1.5 Adjoint and Inverse of the non-singular matrices of order 2 and 3 1.6 Characteristic equation and Eigen values of a square matrix of order 2	10-0-0
UNIT-2 STRAIGHT LINES	➢ Able to find the equation of a straight line in different forms ➢ Determine whether the lines are parallel or perpendicular	2.1 Slope of a straight line 2.2 Intercepts of a straight line 2.3 Intercept form of a straight line 2.4 Slope-intercept form of a straight line 2.5 Slope-point form of a straight line 2.6 Two-point form of a straight line 2.7 General form of a straight line 2.8 Angle between two lines and conditions for lines to be parallel and perpendicular 2.9 Equation of a straight line parallel to the given line 2.10 Equation of a straight line perpendicular to the given line	10-0-0
UNIT-3 TRIGONOMETRY	➢ Use basic trigonometric skills in finding the trigonometric ratios of allied and compound angles ➢ Able to find all the measurable dimensions of a triangle	3.1 Concept of angles, their measurement, Radian measure and related conversions. 3.2 Signs of trigonometric ratios in different quadrants (ASTC rule) 3.3 Trigonometric ratios of allied angles (definition and the table of trigonometric ratios of standard allied angles say $90°\pm\theta$, $180°\pm\theta$, $270°\pm\theta$ and $360°\pm\theta$) 3.4 Trigonometric ratios of compound angles (without proof) 3.5 Trigonometric ratios of multiple angles 3.6 Transformation formulae	10-0-0

Syllabus

As per C-20 D.T.E. Syllabus

UNIT-4 DIFFERENTIAL CALCULUS AND APPLICATIONS	➢ Able to differentiate algebraic, exponential, trigonometric, logarithmic and composite functions ➢ Able to find higher order derivatives ➢ Understand and work with derivatives as rates of change in mathematical models ➢ Find local maxima and minima of a function	4.1 Derivatives of continuous functions in an interval (List of formulae) 4.2 Rules of differentiation 4.3 Successive differentiation (up to second order) 4.4 Applications of differentiation	11-0-0
UNIT-5 INTEGRAL CALCULUS AND APPLICATIONS	➢ Understand the basic rules of integration and Evaluate integrals with basic integrands. ➢ Identify the methods to evaluate integrands ➢ Apply the skills to evaluate integrals representing areas and volumes	5.1 List of standard integrals and Basic rules of integration 5.2 Evaluation of integrals of simple function and their combination 5.3 Methods of integration 5.4 Concept of definite integrals 5.5 Applications of definite integrals	11-0-0

Book Contents

Book is structured according to syllabus

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Unit: 01
Matrices & Determinates
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1
Matrices and Determinants

1.1 BASIC CONCEPTS OF MATRICES

A matrix is a rectangular array of numbers or elements and subjected to certain rules of operations.

Matrix having m rows n columns is of order $m \times n$ and total number of elements in matrix are mn (Read as m cross n or m by n). A matrix is denoted by capital letter.

The elements of the matrix are enclosed within the square brackets [ ].

$$\text{Let} \quad A = \begin{bmatrix} a_{11} & a_{12} \cdots\cdots & a_{1j} \cdots\cdots & a_{1n} \\ a_{21} & a_{22} \cdots\cdots & a_{2j} \cdots\cdots & a_{2n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ a_{i1} & a_{i2} \cdots\cdots & a_{ij} \cdots\cdots & a_{in} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ a_{m1} & a_{m2} \cdots\cdots & a_{mj} \cdots\cdots & a_{mn} \end{bmatrix}$$

The numbers of functions a_{ij} are called the elements of the matrix A. The elements of A are having double subscript notation. The first subscript indicates the row and the second subscript indicates the column.

Illustrations

(i) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $\begin{matrix} \rightarrow 1^{st} \text{ Row} \\ \rightarrow 2^{nd} \text{ Row} \\ \rightarrow 3^{rd} \text{ Row} \end{matrix}$

$\downarrow$ 1st Col. $\downarrow$ 2nd Col. $\downarrow$ 3rd Col. (m × n)

Unit: 01
Matrices & Determinates

where, m = number of rows and n = number of columns,

(ii) $B = \begin{bmatrix} 5 & -1 & 6 \\ 2 & 5 & 8 \end{bmatrix}$ → 1st Row, → 2nd Row; (2 × 3); ↓ 1st Col. ↓ 2nd Col. ↓ 3rd Col.

(iii) $C = \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}_{2\times 2}$

1.2 TYPES OF MATRIX

1. **Row Matrix :** A matrix having only one row is called a row matrix.
   Ex. $A = [8 \ \ -1 \ \ 5]$ is a row matrix of order 1 × 3.
2. **Column Matrix :** A matrix having only one column is called a column matrix.

   Ex. $A = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ is a column matrix of order 3 × 1.

3. **Square Matrix :** A matrix having equal numbers of rows and columns is called a square matrix.

   Ex: $A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}_{2\times 2}$ $B = \begin{bmatrix} 1 & 5 & 8 \\ 3 & -2 & 1 \\ 5 & 8 & 7 \end{bmatrix}_{3\times 3}$

4. **Diagonal Matrix :** A square matrix whose all the elements except the diagonal elements are zero is called a diagonal matrix.

   Ex : $A = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

5. **Scalar Matrix :** It is a diagonal matrix in which all the elements in the principal diagonal are **same**.

   Ex : $A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ is a scalar matrix of order 3 × 3

6. **Unit Matrix or Identity Matrix :** It is denoted by I. It is a square matrix whose all non diagonal elements are zero and diagonal elements are 1 each.

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Matrices & Determinates

Ex : $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}$ $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$

7. **Zero Matrix or Null Matrix :** A matrix whose all the elements are zero is known as zero matrix. It is denoted by 0.

Ex : $0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

8. **Equal Matrix :** The matrices A and B are said to be equal if their orders are same and elements at the corresponding places are equal.

Ex : if $A = \begin{bmatrix} 3 & 4 & 5 \\ 8 & 1 & 2 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 3 & 4 & 5 \\ 8 & 1 & 2 \end{bmatrix}_{2\times3}$ then $A = B$

9. **Symmetric matrix :** A square matrix, A is said to be symmetric if $A' = A$

Ex : $A = \begin{bmatrix} 3 & 4 & -2 \\ 4 & 3 & 1 \\ -2 & 1 & 5 \end{bmatrix}$ then $A' = A$, therefore A is a symmetric.

10. **Skew symmetric matrix :** if $A' = -A$ then A is called skew symmetric matrix.

Ex : $A = \begin{bmatrix} 0 & -4 & 2 \\ 4 & 0 & -8 \\ -2 & 8 & 0 \end{bmatrix}$ then A is a skew symmetric matrix.

1.3 SCALAR MULTIPLICATION

If k is a scalar (i.e., a single number) and A is a matrix of any size, then kA is a matrix obtained by multiplying each elements of A by the scalar k.

Illustration (1)

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then,

$$kA = k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

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Matrices & Determinates

Illustration (2)

$$\text{If} \quad A = \begin{bmatrix} 4 & 2 & 1 \\ 5 & 8 & -2 \end{bmatrix} \text{ then, } \quad 3A = \begin{bmatrix} 4 & 2 & 1 \\ 5 & 8 & -2 \end{bmatrix} = \begin{bmatrix} 12 & 6 & 3 \\ 15 & 24 & -6 \end{bmatrix}$$

Illustration (3)

$$\text{If} \quad A = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} \text{ then,}$$

$$\frac{1}{2}A = \frac{1}{2}\begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{2} & \frac{2}{2} \\ \frac{-1}{2} & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ \frac{-1}{2} & \frac{3}{2} \end{bmatrix}$$

1.4 ADDITION OF MATRICES

If A and B are two matrices of the same order, then their sum $A + B$ is defined and is obtained by adding the corresponding elements of A and B.

Illustration, If $A = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 5 \\ 3 & 1 & 4 \end{bmatrix}$

$$\text{then,} \quad A + B = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 5 \\ 3 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 3+2 & 2+0 & 1+5 \\ 5+3 & 1+1 & 4+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 6 \\ 8 & 2 & 8 \end{bmatrix}$$

1.5 SUBTRACTION OF MATRICES

If A and B are two matrices of the same order, then their difference $A - B$ is defined and is obtained by subtracting the elements of B from the corresponding elements of A.

$$\text{Illustration,} \quad \text{If} \quad A = \begin{bmatrix} 2 & 1 \\ 3 & 8 \\ 9 & 12 \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 & 1 \\ 5 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\text{then,} \quad A - B = \begin{bmatrix} 2 & 1 \\ 3 & 8 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 1 \\ 5 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2-6 & 1-1 \\ 3-5 & 8-3 \\ 9-(-1) & 12-4 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -2 & 5 \\ 10 & 8 \end{bmatrix}$$

Properties of addition of Matrices

If A, B, C and D are matrices of same order. Where, O is the null matrix, then

Unit: 02
Straight Line

Pages: 78-112

2

Straight Line

2.1 STRAIGHT LINES

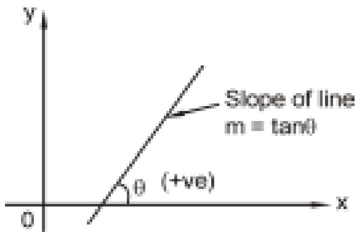
A straight line is an endless, one dimensional figure that has no width. It is a combination of endless points joined on both sides of a point. A straight line does not have any curve in it. It can be horizontal, vertical or inclined.

A straight line is a figure formed when two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are connected between them, and the line ends are extended to infinity.

2.2 SLOPE / GRADIENT OF A LINE

If a straight line makes an angle θ with the positive direction of the x-axis, then θ is called the inclination and tangent angle (tanθ) is called slope (m) or gradient of the line. Slope is often denoted by the letter "m".

Slope of a line is calculated by finding the ratio of change in $Y(\Delta Y)$ to the change in $X(\Delta X)$ between two distinct points on a line.



The slope of a line is generally given as $m = \tan\theta$

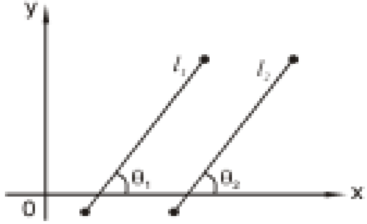
Unit: 02
Straight Line

Note :

1. The slope of a line is positive or negative according to the inclination 'θ' of the line with respect to x-axis.
2. Slope is **positive**, when θ is an acute angle i.e., θ lies between **0° and 90°**.
3. Slope is **negative**, when θ is an obtuse angle i.e., θ lies between **90° and 180°**.
4. Slope is **zero**, when θ is equal to 0° or it is **parallel to x-axis**.
5. Slope is **infinite**, when θ is equal to 90° or it is **perpendicular to x-axis**.

2.3 SLOPES OF PARALLEL LINES

Let l_1 and l_2 be two non-vertical parallel lines with respective inclination θ_1 and θ_2 with the positive direction of the x-axis.



Here l_1 and l_2 be parallel lines. Hence their inclinations of the lines are equal.

$$\therefore \theta_1 = \theta_2$$

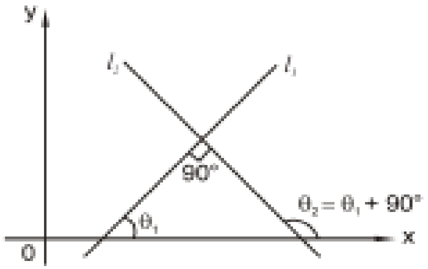
$$\text{then } \tan\theta_1 = \tan\theta_2$$

$$\therefore \boldsymbol{m_1 = m_2} \qquad \because \tan\theta = m$$

Hence slope of two parallel lines are equal.

2.4 SLOPE OF PERPENDICULAR LINES

Let l_1 and l_2 be two non-vertical lines with respective inclination θ_1 and θ_2 with the positive direction of the x-axis.



Here l_1 and l_2 be perpendicular lines. Hence inclination of the line l_2 is

$$\theta_2 = \theta_1 + 90°$$

$$\text{then } \tan\theta_2 = \tan(\theta_1 + 90°)$$

$$\tan\theta_2 = -\cot\theta_1$$

$$\tan\theta_2 = -\frac{1}{\tan\theta_1}$$

$$\tan\theta_1 \cdot \tan\theta_2 = -1$$

$$\therefore \boldsymbol{m_1 \cdot m_2 = -1}$$

Hence slope of two perpendicular lines are product of slope of l_1 and l_2 be minus one.

Unit: 02
Straight Line

Note :

Slope of line having different inclination with x-axis									
(Inclination) Angle (θ)	0°	30°	45°	60°	90°	120°	135°	150°	180°
Slope (m)	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	–1	$-\frac{1}{\sqrt{3}}$	0

EXAMPLE PROBLEMS

1. What is the slope of a line whose inclination is 120°.

Solution : Given $\theta = 120°$

$\therefore$ slope, $m = \tan\theta = \tan 120° = \tan(180° - 60°)$

$$m = -\tan 60°$$

$$m = -\sqrt{3}$$

2. What is the slope of a line whose inclination is 45°.

Solution : Given $\theta = 45°$

$\therefore$ slope, $m = \tan\theta = \tan 45°$

$$m = 1$$

3. Find the slope of the line joining the points (2, –3) (6, 1).

Solution : Given $P(x_1, y_1) = (2, -3)$ and $Q(x_2, y_2) = (6, 1)$

$$\text{The slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-(-3)}{6-2} = \frac{1+3}{4} = \frac{4}{4}$$

$$\therefore m = 1$$

4. Find the slope of the line parallel to the line joining the points (2, 3) (4, –1).

Solution : Given $A(x_1, y_1) = (2, 3)$ and $B(x_2, y_2) = (4, -1)$

$$\text{Slope of line, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{4-2} = \frac{-4}{2} = -2$$

w.k.t slope of the parallel lines are equal.

$$m_1 = m_2 = m = -2$$

$\therefore$ Slope of parallel line is –2

Unit: 03
Trigonometry

Pages: 113-190

3

Trigonometry

The word Trigonometry is derived from the Greek word "trigon", meaning the three sides i.e., the triangle and "metron" means measure. So, trigonometry is study of relationship between the sides and the angle of a triangle.

Trigonometry is a branch of mathematics that deals with the study of the relationship between the sides and angle of a right angled triangle.

3.1 CONCEPT OF ANGLES AND MEASUREMENT

Angle can be defined as the rotation from the initial point to an endpoint of a ray.

In geometry, an angle measure can be defined as the measure of the angle formed by the two rays or arms at a common vertex.

The concepts of equality, sums and differences of angles are important and used throughout geometry, but the topic of trigonometry is based on the measurement of angles.

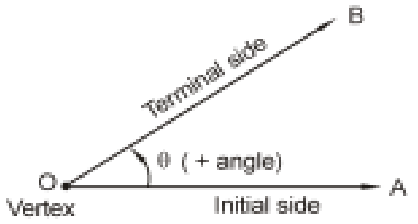
Commonly used terms in describing the angles are:

Initial side : The original ray

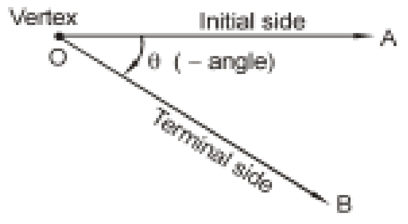
Terminal side : The final position of the ray after rotation

Vertex : Point of rotation

Positive angle : The direction of rotation is anticlockwise, then the angle is called positive angle.



(i) Positive angle



(ii) Negative angle

Unit: 03
Trigonometry

Negative angle : The direction of rotation is clockwise, then the angle is called negative angle.

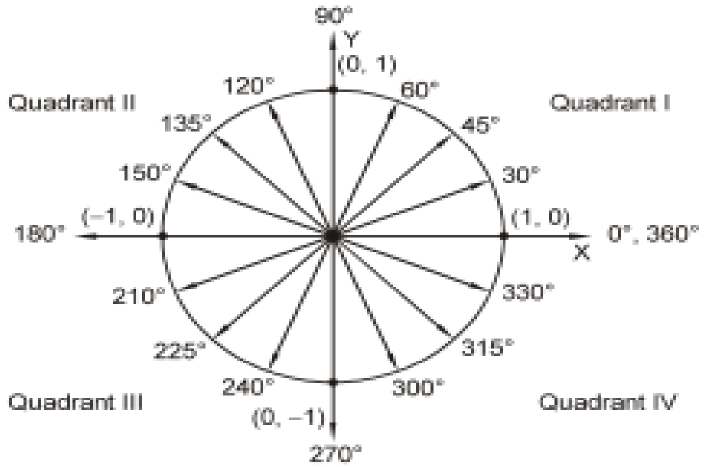
The two commonly used units for measurement of angles are :

i) Degree measurement

ii) Radian measurement

1. Angle measurement - Degree Measure

A complete revolution, i.e., when the initial and terminal sides are in the same position after rotating clockwise, is divided into 360° units called degrees. So, if the rotation from the initial side to the terminal side is $\left(\frac{1}{360}\right)^{th}$ of a revolution, the angle is said to be one degree. It is denoted as 1°



Time is measured in hours, minutes and seconds, where, 1 hour = 60 minutes and 1 minute = 60 seconds, similarly, while measuring angles,

- 1 degree = 60 minutes, denoted as $1° = 60'$
- 1 minute = 60 seconds, denoted as $1' = 60''$

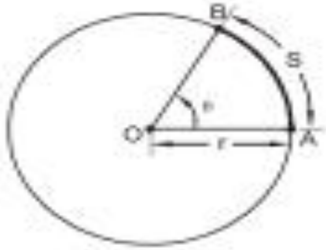
2. Angle measurement - Radian Measure

Radian is an angle subtended at the center of a circle by an arc, whose length of arc (s) is equal to the radius (r) of the circle.

Hence, length of arc (S) = radius of circle (r)

then $\angle AOB$ = 1 Radian

Unit: 03
Trigonometry

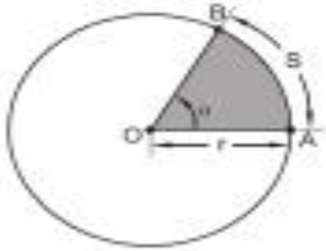


Let "S" be the length of the arc of a circle.

'θ' radians in the angle subtended by the arc at the centre and r is the radius of the circle then, arc length $S = r\theta$

7. Area of sector

The space enclosed by the sector of a circle is called the area of sector. A sector always originates from the circle. The area of sector formed by an angle θ it the centre of circle of radius r.



$$\textbf{Area of sector} = \frac{1}{2} r^2 \theta$$

EXAMPLE PROBLEMS

1. Convert 60° into radians.

Solution : Given angle $\theta = 60°$

$$\text{Angle in radian} = \frac{\pi}{180} \times \theta = \frac{\pi}{180} \times 60° = \frac{\pi}{3} \text{ radian} = 1.047 \text{ radian}$$

2. Express 75° into radians. **(Nov - 2012)**

Solution : Given angle $\theta = 75°$

$$\text{Angle in radian} = \frac{\pi}{180} \times \theta = \frac{\pi}{180°} \times 75° = 0.4167\pi = 1.309 \text{ radian}$$

Unit: 04

Differential Calculus & It's Applications

Pages: 191-308

4

Differential Calculus and It's Applications

4.1 INTRODUCTION

Differential calculus is one of the most important break throughs in modern - mathematics, science and engineering. In these days it is desirable to know how a particular parameter is changing with respect to some other parameter. The derivative of a function measures the sensitivity of one variable to small change in other variable. Differential calculus determines the instantaneous speed and acceleration at any given specific instant of time and is useful in finding the slope of tangent and normal to a curve at a certain point. In the present chapter we shall introduce the concept of derivative of a function, process of differentiation of different forms of functions and some of the applications.

4.2 DERIVATIVE OF A FUNCTION

Let $y = f(x)$ be a real valued function defined on the interval I. Let the variable x be changed from x to $x + \delta x$ then the corresponding change in y be from y to $y + \delta y$.

We have $y = f(x)$

After changing $x \to x + \delta x$, $y \to y + \delta y$,

$$y + \delta y = f(x + \delta x)$$

$$\delta y = f(x + \delta x) - y$$

$$\delta y = f(x + \delta x) - f(x) \qquad \because y = f(x)$$

$$\frac{\delta y}{\delta x} = \frac{f(x+\delta x) - f(x)}{\delta x}$$

Taking the limit as $\delta x \to 0$ on both sides

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

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Differential Calculus & It's Applications

If the RHS limit exists and finite we say that the function $y = f(x)$ is differentiable at x and that limit is called as derivative of $y = f(x)$ wrt 'x'.

The derivative of a function $y = f(x)$ is denoted by $\frac{dy}{dx}$ or $\frac{d}{dx}(f(x))$ or $f'(x)$

Definition : Let $y = f(x)$ be a function defined in the interval I. Then the derivative of $y = f(x)$ wrt 'x' is defined as

$$\frac{dy}{dx} = f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x} \quad \text{---- (1)}$$

If we denote the small increment δx by h then equation (1) becomes

$$\frac{dy}{dx} = f'(x) = \lim_{\delta x \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{---- (2)}$$

4.3 DIFFERENTIATION FROM THE FIRST PRINCIPLE

The process of obtaining the derivative of a function using the definition is called the differentiation from the first principle.

1. Differentiate a constant function wrt 'x' from first principle.

Solution : Let $f(x) = k$, $f(x + h) = k$

By the definition, we have

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{k - k}{h} = \lim_{h \to 0} \frac{o}{h} = \lim_{h \to 0} 0 = 0$$

i.e., $\frac{d}{dx}(k) = 0$, k is any constant

2. Differentiate x^n wrt 'x' from first principle.

Solution : Let $y = f(x) = x^n$, $f(x + h) = (x + h)^n$

By the definition, we have

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

Unit: 04
Differential Calculus & It's Applications

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{(x+h) - x}$$

As $h \to 0$, $x + h \to x$

$$\frac{dy}{dx} = n \cdot x^{n-1} \qquad \left| \because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1} \right.$$

i.e., $\frac{d}{dx}(x^n) = n\,x^{n-1}$

3. Differentiate $\sqrt{x}$ wrt 'x' from first principle.

Solution : Let $y = f(x) = \sqrt{x}$, $f(x+h) = \sqrt{x+h}$

By the definition, we have

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \text{ (Rationalizing)}$$

$$= \lim_{h \to 0} \frac{\left(\sqrt{x+h}\right)^2 - \left(\sqrt{x}\right)^2}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{x + h - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

i.e., $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

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5

Integral Calculus and it's Applications

5.1 INTRODUCTION

In this chapter we shall introduce the concept of integration of a function as an inverse process of differentiation and see the different methods of integration. The concept of integration is useful in finding the areas of plane regions and volume of surface generated by the curve. The term integration signifies "summation".

5.2 INDEFINITE INTEGRAL

Let $y = f(x)$ be a function. If $F(x)$ is the differential co-efficient of $f(x)$ then $f(x)$ itself is called as an integral, a primitive or anti-derivative of $F(x)$ w.r.t. 'x'

It is denoted by $\int F(x)\ dx = f(x)$

ie., $\int F(x)\ dx = f(x) \Leftrightarrow F(x) = f'(x)$

Since the derivative of a constant is zero i.e., $\frac{d}{dx}\ (f(x) + c) = \frac{d}{dx}\ (f(x)) = F(x)$

$\Rightarrow \int F(x)\, dx = f(x) + C$

where 'C' is called a constant of integration.

In $\int F(x)\, dx$,

$\int$ $\rightarrow$ integral sign (elongated s)

$F(x) \rightarrow$ Integrand, $dx \rightarrow$ w.r.t. 'x'

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Illustrations

1. $\frac{d}{dx}(x) = 1 \Leftrightarrow \int 1 \; dx = x + c$
2. $\frac{d}{dx}(x^3) = 3x^2 \Leftrightarrow \int 3x^2 \; dx = x^3 + c$
3. $\frac{d}{dx}(\tan x) = \sec^2 x \Leftrightarrow \int \sec^2 x \; dx = \tan x + c$
4. $\frac{d}{dx}(\log x) = \frac{1}{x} \Leftrightarrow \int \frac{1}{x} \; dx = \log x + c$

We already know the formulae for the derivatives of some standard functions. From these formulae, we can write down the corresponding formulae for the integrals of these functions.

5.3 LIST OF STANDARD INTEGRALS

1. $\int x^n \; dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$

Illustrations

(i) $\int 1 \; dx = x$ (ii) $\int x \; dx = \frac{x^2}{2}$ (iii) $\int x^2 dx = \frac{x^3}{3}$

(iv) $\int \sqrt{x} \; dx = \frac{x^{3/2}}{3/2}$ (v) $\int x^{3/2} \; dx = \frac{x^{5/2}}{5/2}$

2. $\int \frac{1}{\sqrt{x}} \; dx = 2\sqrt{x} + C$
3. $\int \frac{1}{x} \; dx = \log x + C$
4. $\int \frac{1}{x^2} \; dx = -\frac{1}{x} + C$
5. $\int a^x \; dx = \frac{a^x}{\log a} + C$
6. $\int e^x \; dx = e^x + C$
7. $\int \sin x \; dx = -\cos x + C$
8. $\int \cos x \; dx = \sin x + C$
9. $\int \sec^2 x \; dx = \tan x + C$
10. $\int \operatorname{cosec}^2 x \; dx = -\cot x + C$
11. $\int \sec x \, . \tan x \; dx = \sec x + C$
12. $\int \operatorname{cosec} x \, . \cot x \; dx = -\operatorname{cosec} x + C$
13. $\int \frac{1}{\sqrt{1-x^2}} \; dx = \sin^{-1} x + C$

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14. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C$

15. $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$

16. $\int \sinh x\,dx = \cosh x + C$

17. $\int \cosh x\,dx = \sinh x + C$

18. $\int \operatorname{sech}^2 x\,dx = \tanh x + C$

19. $\int \operatorname{cosech}^2 x\,dx = -\coth x$

20. $\int \operatorname{sech} x.\tanh x\,dx = -\operatorname{sech} x + C$

21. $\int \operatorname{cosech} x.\coth x\,dx = -\operatorname{cosech} x + C$

21. $\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}x$

22. $\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1}x$

Note : In the above set of standard integrals if 'x' is replaced by a linear expression $ax+b$, the same formula holds good with a factor $\frac{1}{a}$ on the right side.

Illustrations

1. $\int \sin(2x+3)\,dx = \frac{-\cos(2x+3)}{2} + C$

2. $\int \sec^2(7-4x)\,dx = \frac{\tan(7-4x)}{-4} + C$

3. $\int e^{4x-1}\,dx = \frac{e^{4x-1}}{4} + C$

4. $\int \frac{1}{5x-4}\,dx = \frac{\log(5x-4)}{5} + C$

5. $\int \frac{1}{\sqrt{1-(4x)^2}}\,dx = \frac{\sin^{-1}(4x)}{4} + C$

5.4 BASIC RULES OF INTEGRATION

1. If K is any non-zero constant then $\int K.f(x)\,dx = K\int f(x)\,dx$

Illustrations

1. $\int 4x\,dx = 4\int x\,dx = 4\,\frac{x^2}{2} = 2x^2 + C$

2. $\int 2.\frac{1}{\sqrt{x}}\,dx = 2\int \frac{1}{\sqrt{x}}\,dx = 2.2\sqrt{x} = 4\sqrt{x} + C$

3. $\int 3\sec^2 x\,dx = 3\int \sec^2 x\,dx = 3\tan x + C = 3\tan x + C$

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2. If $f_1(x)$ and $f_2(x_2)$ are any two functions then

$$\int (f_1(x) \pm f_2(x))\, dx = \int f_1(x)\, dx \pm f_2(x)\, dx$$

In general, by combining both the rules, we can write

$$\int [k_1 f_1(x) \pm k_2 f_2(x) \pm \ldots \pm f_n(x)]\, dx = k_1 \int f_1(x)\, dx \pm k_2 \int f_2(x)\, dx \pm \ldots + k_n \int f_n(x)\, dx$$

Illustrations :

1. $\int \left(x^2 + \frac{1}{\sqrt{x}} \right) dx = \int x^2\, dx + \int \frac{1}{\sqrt{x}}\, dx = \frac{x^3}{3} + 2\sqrt{x} + C$

2. $\int (\sin x + \cos x)\, dx = \int \sin x\, dx + \int \cos x\, dx = -\cos x + \sin x + C$

3. $\int \left(x^3 + \sec^2 x + \frac{1}{1+x^2} \right) dx = \int x^3\, dx + \int \sec^2 x\, dx + \int \frac{1}{1+x^2}\, dx$

$= \frac{x^4}{4} + \tan x + \tan^{-1} x + C$

EXAMPLE PROBLEMS

1. Evaluate $\int (x^2 + 4x + 1)\, dx$

Solution : Let $I = \int (x^2 + 4x + 1)\, dx$

$= \int x^2 dx + 4\int x\, dx + \int 1\, dx$

$= \frac{x^3}{3} + 4.\frac{x^2}{2} + x + C = \frac{x^3}{3} + 2x^2 + x + C$

2. Evaluate $\int \left(3e^x + \frac{4}{x} + 6 \right) dx$

Solution : Let $I = \int \left(3e^x + \frac{4}{x} + 6 \right) dx$

$= \int 3e^x\, dx + \int 4 \cdot \frac{1}{x}\, dx + \int 6 \cdot dx$

Model Question Paper

Pages: 395-397

CODE : 20SC01T

MODEL QUESTION PAPER – 2020

ENGINEERING MATHEMATICS

Time : 3 Hours] [Max. Marks : 100

Instruction: Answer one full question from each section. One full question carries 20 marks.

Section - 1

1. (a) If the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 2 & x & 2 \\ 6 & 8 & 14 \end{bmatrix}$ is singular then find x. **4**

   (b) Find the A^2 for the matrix $\begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}$ **5**

   (c) Solve $2x - y = 3$ and $x + 2y = 4$ by using determinant method. **5**

   (d) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 1 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$ **6**

2. (a) If $A = \begin{bmatrix} 2 & -1 \\ 4 & 0 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 & 4 \\ -1 & -1 & 1 \\ 0 & 4 & 2 \end{bmatrix}$ then find $(AB)^T$. **4**

   (b) Verify whether $AB = BA$ for the matrices, $A = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 4 \\ 0 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix}$.

   (c) Find the Adjoint of the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 2 & 1 \\ 0 & 3 & 3 \end{bmatrix}$. **5**

Model Question Paper

Pages: 392-394

CODE : 20SC01T

I SEMESTER DIPLOMA EXAMINATION, APRIL / MAY - 2021

ENGINEERING MATHEMATICS

Time : 3 Hours] [Max. Marks : 100

Instructions : (i) Answer **one** full question from each section

(ii) **One** full question carries **20** marks

Section - I

1. (a) Find the value of x, if $\begin{vmatrix} 1 & 2 & 9 \\ 2 & x & 0 \\ 3 & 7 & -6 \end{vmatrix} = 0$ **4**

(b) If $A = \begin{bmatrix} 2 & 1 \\ 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$ find AB. **5**

(c) Solve the equations $x + y = 0$, $y + z = 1$ and $x + z = 3$ for y by Cramer's rule. **5**

(d) If $A = \begin{bmatrix} 3 & 1 & 2 \\ -2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$ find A^{-1}. **6**

2. (a) Evaluate $\begin{vmatrix} 2 & 3 & -1 \\ 3 & -2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$ **4**

(b) If $A = \begin{pmatrix} -1 & 0 \\ 5 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 2 & 4 \end{pmatrix}$ prove that adj (AB) = [adj(B) adj(A)]. **5**

(c) Verify whether $AB = BA$ for the matrices.

$A = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 & 4 \\ 0 & -1 & 1 \\ 2 & 4 & -2 \end{bmatrix}$ **5**

(d) Find the characteristic equation and eigen values for the matrix $\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$. **6**

Model Question Paper

Pages: 398-400

CODE : 20SC01T

I SEMESTER DIPLOMA EXAMINATION, OCT. / NOV. - 2021

ENGINEERING MATHEMATICS

Time : 3 Hours] [Max. Marks : 100

Special Note : Students can answer for max. of 100 marks, selecting any sub-section from any main section

Section - I

1\. (a) If the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 4 \\ 3 & 2 & x \end{bmatrix}$ is singular, then find 'x'. **4**

(b) If $A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, then find $2A + 3B$. **5**

(c) Solve the system of equations $2x + 3y = 5$ and $x + 4y = 5$ by Carmer's Rule. **5**

(d) Find the characteristic roots for the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$. **6**

2\. (a) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 3 & 6 \end{bmatrix}$, then find $(A + B)^2$. **4**

(b) If $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 0 \\ 4 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 1 & 3 & -2 \end{bmatrix}$, then find $(AB)^T$. **5**

(c) Find the characteristic equation and Eigen roots of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. **5**

(d) Find the Inverse of the matrix $A = \begin{bmatrix} 5 & 1 & 3 \\ 1 & 4 & -1 \\ 2 & 0 & 4 \end{bmatrix}$. **6**

Book Back Cover Page

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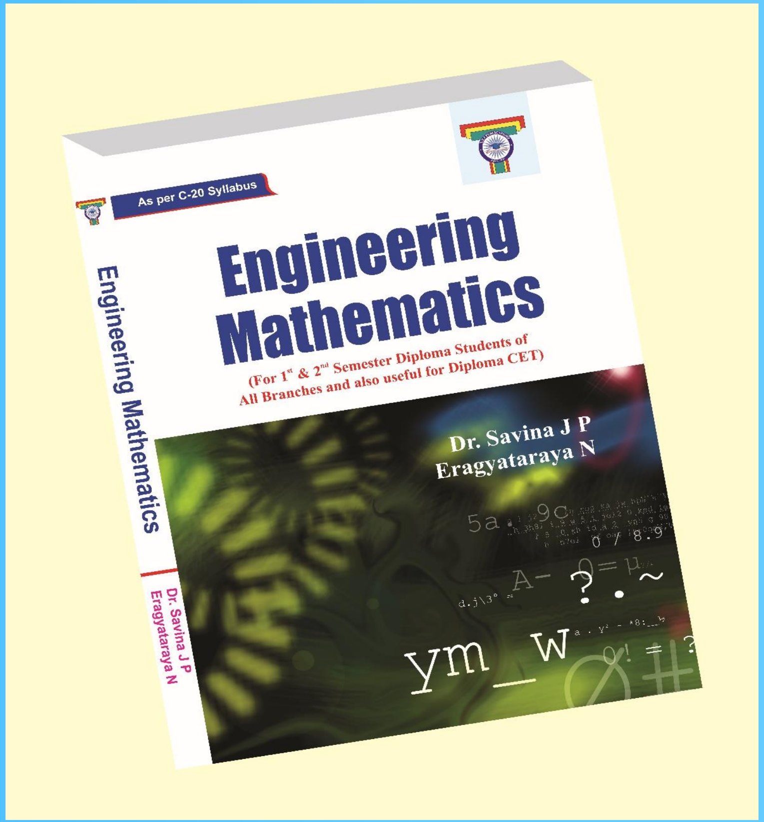

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